RESEARCH STATEMENT

My work is focused on problems in nonlinear PDE theory arising from geophysics, fluid dynamics, biology and material science. A significant portion of my work is devoted to local and global issues related to free boundary problems. Three subjects are being investigated regarding this aspect: fluid-structure interactions, primitive equations and Euler equations. Fluid-structure interactions are outstanding problems involving free boundaries separating different types of equations (hyperbolic and parabolic) representing the interaction between elastic and fluid media. The complexities involved in the interaction between different types of media which influence each other across a shared interface are interesting and challenging. I worked mainly on the problem of elastic bodies immersed in an incompressible fluid. The elastic body deformations obey linear elasticity equations. The fluid obeys the incompressible Navier-Stokes equations. The boundary conditions impose continuity of velocities and normal stresses at the interface. The different mathematical properties of hyperbolic and parabolic equations manifest themselves in a regularity mismatch. The nonlinear equations and the Lagrangian approach require a number of derivatives to be well behaved. In collaboration with I. Kukavica, I. Lasiecka and A. Tuffaha, I have established in [21] local well posedness in Sobolev spaces for general data. In [22, 23], I obtained the first global existence results for the system when damping is added.

The primitive equations are widely used models of ocean dynamics. In collaboration with I. Kukavica and M. Ziane, we established in [26] the well posedness of the free boundary problem for the primitive equations of the ocean. We constructed a local in time, unique, real analytic solution and gave an explicit rate of decay for the radius of analyticity. This provided a solution to a problem (cf. [35]) which has been open since 1994.

Another area of my research concerns the Prandtl equations and the inviscid limit. Here I aim to make progress in the understanding of the asymptotic description of high Reynolds numbers flows in the presence of boundaries. The Prandtl equations arise in a classical asymptotic expansion of Navier-Stokes equations for wall bounded flows, in the limit of vanishing viscosity. Together with V. Vicol I have obtained almost global existence of small solutions in anisotropic Sobolev-analytic spaces [27]. “Almost global” refers here to solutions whose lifespan is exponential in ϵ^{-1} when the size of the data is of order ϵ. This greatly improved previous results which are algebraic (of order $\epsilon^{-\frac{4}{3}}$). In work with P. Constantin, T. Elgindi and V. Vicol, I obtained sufficient conditions for the strong convergence of solutions of Navier-Stokes equations with no-slip boundary conditions to a smooth solution of the Euler equation (with non-penetrative boundary conditions) in a half-space [6]. The conditions require equicontinuity of the product of velocities at the boundary but do not involve derivatives.

A third area of my research concerns nonlocal evolution equations in bounded domains. Here I investigate regularity properties of solutions of the surface quasi-geostrophic equation and of models of electroconvection. For the latter, with P. Constantin, T. Elgindi and V. Vicol we established global regularity and uniqueness of solutions with arbitrary large data [5]. The model we considered is relevant for a specific set of physical experiments in which a charged fluid film was driven by voltage differences applied at its boundaries. The model consisted of the two-dimensional Navier-Stokes equations driven by electrical forces, coupled to an equation of evolution of the charge density which was advected by the Navier-Stokes flow and dissipated via a Dirichlet-to-Neumann operator. Regarding the SQG, with P. Constantin, I obtained in [7, 8] global a priori interior

Lipschitz bounds for the critical SQG in bounded domains, when the fractional Laplacian and Riesz transforms correspond to the Laplacian with homogeneous Dirichlet boundary conditions.

I also worked on Harnack inequalities for elliptic and parabolic equations with supercritical divergence-free drift [24, 25, 16], and on quantitative unique continuation and estimates of complexity of solutions for higher order elliptic and parabolic equations [17, 18, 19].

A more detailed description of my current work follows below.

1. Free boundary value problems

1.1. **Free boundary fluid-structure models – local and global in time well-posedness.**

Fluid-structure interaction is the name used to describe problems in which an elastic material interacts with an internal or surrounding fluid. This is very interesting mathematically because the coupling between the fluid dynamical description in terms of velocity, pressure, and the elasticity description in terms of deformation, stress, is mediated by the interface between the two phases. The mismatch of regularity between hyperbolic and parabolic dynamics constitutes a defining feature of the problem. Fluid-structure interaction models have broad applications and are studied in several fields: aeronautical, naval, structural and mechanical engineering (from off-shore structures to sport applications), biomedicine (interaction of blood with arterial vessels), applied biology (motility).

We consider the scenario where the elastic material is fully immersed in the fluid. The problem setting is as follows. Let $\Omega \subset \mathbb{R}^3$ be a fixed open, bounded, connected and smooth domain which represents the fluid container in which both the elastic solid and the fluid move. Initially Ω consists of an exterior region Ω_f for the fluid and an interior region Ω_e for the structure, with a common boundary Γ_c. The initial configurations Ω_f and Ω_e evolve in time under the flow map $\eta(\cdot,t)\colon \Omega \to \Omega$ such that $\Omega_f(t) = \eta(\Omega_f, t)$ represents the domain occupied by the fluid at time t and $\Omega_e(t) = \eta(\Omega_e, t)$ is the domain occupied by the elastic structure at time t, with interface $\Gamma_c(t) = \overline{\Omega_e(t)} \cap \overline{\Omega_f(t)}$ between the fluid and the structure at time t.

The equations of motion for incompressible fluids are governed by the Navier-Stokes equations, posed in the Eulerian framework for the fluid velocity field $u(x,t)$ and the scalar pressure $p(x,t)$:

$$\rho_f(u_t + (u\cdot\nabla)u) = \nabla\sigma(u,p) + \rho_f F_f \text{ in } \Omega_f(t)$$
$$\nabla u = 0 \text{ in } \Omega_f(t)$$

where we denoted ρ_f the fluid density, $\nu > 0$ the kinematic viscosity, $\sigma(u,p) = -pI + \nu\varepsilon(u)$ the Cauchy stress tensor, $\varepsilon(u) = 1/2(\nabla u + \nabla u^T)$ the deformation tensor, and F_f the body forces acting on the fluid per unit volume.

The elastic body motion (for small deformations) is modeled by linear elasticity (Kirchhoff law), posed in the Lagrangian framework for the elastic displacement $w(x,t) = \eta(x,t) - x$:

$$\rho_e(0) w_{tt} = \nabla\sigma(w) + F_e \text{ in } \Omega_e$$

where $\rho_e(0)$ is the initial density, $\sigma(w) = 2\mu\epsilon(w) + \lambda\nabla w\, I$ the first Piola tensor with Lamé coefficients of the elastic material $\lambda \geq 0$ and $\mu > 0$, $\epsilon(w) = \frac{1}{2}(\nabla w + \nabla w^T)$ the strain tensor, and F_e the body force.

The equations are coupled through natural velocity and stress matching conditions on the moving interface between the fluid and the elastic body. Namely, we assume perfect adherence of the fluid to the structure and the container

$$u \circ \eta = w_t \text{ on } \Gamma_c$$
$$u = 0 \text{ on } \Gamma_f$$

and by the principle of action and reaction

$$\sigma(w)N = [\sigma(u,p) \circ \eta][(\nabla\eta)^{-1}N] \text{ on } \Gamma_c,$$

where N is the outward unit normal to Γ_c. The problem is supplemented by compatible initial conditions $u(x,0) = u_0(x)$ on Ω_f, $w(x,0) = 0$ on Ω_e (no initial deformation), $w_t(x,0) = w_1(x)$ on Ω_e.

In [21], we proved local a priori estimates and uniqueness of the system with initial data $(u_0, w_1) \in H^4 \times H^2$. In [22] and [23], we addressed the global well-posedness for the free boundary fluid-structure system subjected to interior damping. Our paper [22] is the first work where both a global well-posedness theorem and construction of solutions of the problem are established. We considered the system formulated entirely in the Lagrangian framework:

$$\begin{aligned}
&v^i_t - \partial_j(a^j_l a^k_l \partial_k v^i) + \partial_k(a^k_i q) = 0 \text{ in } \Omega_f \times (0,T),\ i = 1,2,3\\
&a^k_i \partial_k v^i = 0 \text{ in } \Omega_f \times (0,T),\\
&w^i_{tt} - \Delta w^i + \alpha w^i_t + \beta w^i = 0 \text{ in } \Omega_e \times (0,T),\ i = 1,2,3,
\end{aligned}$$

where $v(x,t)$ and $q(x,t)$ denote the Lagrangian velocity and the pressure of the fluid over the initial domain Ω_f, while $w(x,t) = \eta(x,t) - x$ denote the structure displacement over the initial domain Ω_e. The parameters $\alpha \geq 0$ and $\beta \geq 0$ are real numbers. The coefficients matrix a, which obeys $a(x,t) = (\nabla\eta(x,t))^{-1}$ in Ω_f, and the flow map η are determined from

$$\begin{aligned}
&a_t = -a : \nabla v : a \text{ in } \Omega_f \times (0,T)\\
&\eta_t = v \text{ in } \Omega_f \times (0,T).
\end{aligned}$$

On Γ_c, we assume continuity of the velocities ($\gamma = 0$) or transmission boundary condition ($\gamma > 0$)

$$w^i_t = v^i - \gamma \partial_j w^i N_j \text{ on } \Gamma_c \times (0,T),$$

and the matching of stresses

$$\partial_j w^i N_j = a^j_l a^k_l \partial_k v^i N_j - a^k_i q N_k \text{ on } \Gamma_c \times (0,T),$$

while on the outside fluid boundary Γ_f we assume the non-slip boundary condition

$$v^i = 0 \text{ on } \Gamma_f \times (0,T)$$

for $i = 1,2,3$, where N is the unit outward normal with respect to Ω_e.

Global existence for small data in quasilinear dynamics is strongly tied to the asymptotic decay of solutions, which in turn requires a dispersive or a damping mechanism. In the case of fluid-structure interaction, the minimal structural damping necessary for decay in the linear case is the interior damping [1]. In [22] we showed that the presence of interior damping ($\alpha > 0$), static damping ($\beta > 0$) and the boundary damping ($\gamma > 0$) allow to prove global existence of solutions to the quasilinear model. In this work it was also shown that the interior damping requirement can be relaxed ($\alpha = 0$) if the elastic body is assumed to be initially star-shaped about a point in its interior.

In [23], we obtained the global existence and decay of the energy without boundary damping ($\gamma = 0$) and with no additional geometric condition for the elastic body. When proving well-posedness of the quasilinear system one is faced with a twofold task: (i) obtaining suitable a priori estimates which close at some desired level of regularity, and (ii) construction of actual solutions to which the a priori estimates can be applied. In order to remove the boundary stabilization condition and allow $\gamma = 0$, both tasks met new substantial challenges which were not previously encountered in [22].

1.2. **Primitive equations and related models.** The primitive equations are a system of nonlinear partial differential equations (PDE) that form the basis of most atmospheric and atmosphere-ocean models used today. The free boundary value problem for the ocean model, introduced by Lions et al. (cf. [35]) is governed by the primitive equations, which are derived from the full compressible Navier-Stokes/Euler equations under the Boussinesq and the hydrostatic balance approximations. The model consists of the conservation equation for the ocean

$$\partial_t v + v\partial_x v + w\partial_z v + \partial_x p = 0,$$

the diagnostic equations

$$\begin{aligned} &\partial_x v + \partial_z w = 0, \\ &\partial_z p + g\rho = 0, \end{aligned}$$

and the evolution equation for the ocean surface

$$\partial_t \zeta + v\partial_x \zeta = w|_{z=\zeta}.$$

The unknown functions are the horizontal and the vertical velocity $v(x, z, t)$ and $w(x, z, t)$, the scalar pressure $p(x, z, t)$, and the position of the ocean surface $z = \zeta(x, t)$ with respect to the level $z = 0$. The constant density ρ, the ocean surface pressure $p(x, \zeta, t)$, and the gravitational constant g are given. Temperature and salinity variations are not included in this simplified version of the model.

In [26], we established the local well-posedness of the free boundary value problem for the ocean with analytic initial data. We included the nonlinear dependence of the vertical velocity on the other unknowns and the pressure term in the evolution equation for the horizontal velocity. We introduced new real-analytic norms, which allowed us to distinguish between the derivatives taken in the horizontal and the vertical directions of the moving domain. In order to close the a priori estimate for the evolution equation for the ocean surface, we provided a combinatorial result and a Faà di Bruno type formula (cf. [26, Section 3]). Using classical Picard's iteration procedure, we constructed a local-in-time, unique, real analytic solution and gave an explicit rate of decay on the radius of analyticity.

1.3. **Free-surface Euler equation with surface tension.** Free boundary problems for Euler and Navier-Stokes equations arise broadly in the study of water waves and of interfaces between immiscible fluids. The question of local well-posedness of the free-surface Euler equations with or without surface tension has been of paramount interest for the past four decades (cf. [33, 34, 12, 13, 44, 43, 49] and the references therein).

In [20], we considered the local well-posedness of solutions in Sobolev spaces to the 3D free-boundary incompressible Euler equations

$$\begin{aligned} &\partial_t u + (u \cdot \nabla) u + \nabla p = 0 \text{ in } \Omega(t), \\ &\nabla \cdot u = 0 \text{ in } \Omega(t), \end{aligned}$$

where the free boundary $\partial\Omega(t)$ evolves according to the fluid velocity field $u(x, t)$, and the pressure obeys

$$p(x, t) = \sigma H \text{ on } \partial\Omega(t).$$

Here $\sigma > 0$ is the surface tension, while H is twice the mean curvature of the boundary $\partial\Omega(t)$. The solutions may contain nontrivial vorticity. In our work [20], we established a priori estimates with the initial velocity in low regularity spaces $H^{3.5}$. Even though the surface tension provides a regularizing mechanism, it also creates many technical challenges. Our approach is based on the

Cauchy invariant (which provides a very natural and economical starting point for the analysis), new pressure estimates, and the structure of the equations.

2. Prandtl equations and the inviscid limit

2.1. **Prandtl equations with small data.** The Prandtl equations are part of an asymptotic description of the Navier-Stokes equations for wall bounded flow in the limit of zero kinematic viscosity, $\nu \to 0$. They represent the evolution of the fluid in a boundary layer of size $\sqrt{\nu}$. The boundary layer appears because the Navier-Stokes equations have velocities which vanish at the wall, while the limiting zero-viscosity equations, the incompressible Euler equations have only non-penetrative boundary conditions for the velocity.

The Prandtl equations are not well-posed in Sobolev spaces, and the existence of solutions is usuallyy considered for short times, in either purely analytic or anisotropic analytic functional settings. The long time existence of solutions of the Prandtl equations has only been considered in classical work of Oleinik [41], and in [48], and [50]. For small initial velocities that are tangentially real-analytic and of size $\mathcal{O}(\epsilon)$, Zhang and Zhang [50] proved that the Prandtl system has a unique solution on a time interval of order $\mathcal{O}(\epsilon^{-4/3})$. Their proof relied on anisotropic Littlewood-Paley energy estimates in tangentially analytic norms. In [27] we showed that the lifespan is much longer, exponentially so, by using a different approach, based on a linearly good unknown function (inspired by [39]). We proved that in 2D the solution exists almost globally in time, i.e. it lives up to time $\mathcal{O}(\exp(\epsilon^{-1}/\log \epsilon^{-1}))$. Our initial datum consists of a stable finite boundary layer lift profile, and an $\mathcal{O}(\epsilon)$ possibly unstable, but tangentially real-analytic profile. In particular, the total initial vorticity is not necessarily positive (i.e., it does not satisfy the Oleinik monotonicity condition).

2.2. **Inviscid limit.** The question of vanishing viscosity behavior of wall bounded flows, where the Navier-Stokes solutions obey homogeneous Dirichlet boundary conditions is classical and fundamental. An interesting result of Kato [28] stated that the strong convergence for short time, as viscosity tends to zero to a smooth solution u^E of the Euler equation holds if, and only if the rate of dissipation of kinetic energy vanishes in a $\mathcal{O}(\nu)$ boundary layer. Kato's result is a stability result of the Euler path $S^E(t)u_0$, conditioned on assumptions on the viscous dissipation at the boundary. There is a large literature concerned with related or similar conditional strong L^2 convergence results (a few examples are [2, 3, 6, 9, 29, 30, 45, 47]). Some strong L^2 unconditional convergence results for short time do exist. They are based on assumptions of real analytic data [42], or the vanishing of the Eulerian initial vorticity in a neighborhood of the boundary [38]. Symmetries can also lead to strong inviscid limits [15, 31, 36, 37, 40]. All these unconditional results are for short time, close to a smooth solution of Euler equation in laminar situations where energy dissipation rates vanish in the limit. The vast majority of the conditional results are also for short time, close to a smooth solution of Euler equation in laminar situations where energy dissipation rates vanish in the limit, and the conditions involve some uniform property of the Navier-Stokes solutions near the boundary such as bounds on derivatives (like the wall shear stress).

Kato's proof is based on the introduction of a boundary layer corrector. In [6] we established sufficient conditions for the strong inviscid limit to hold, without relying on any assumptions concerning derivatives of the Navier-Stokes equations, in the half space $\mathbb{H} = \{(x_1, x_2) \colon x_2 > 0\}$. Specifically, we assumed that the product

$$\{u_1^{\mathrm{NS}} u_2^{\mathrm{NS}}\}_{\nu \in (0,\nu_0]} \quad \text{is equicontinuous at} \quad x_2 = 0.$$

The assumption is sufficient to deduce strong convergence in the same class as the result of Kato: short time strong $L^\infty(L^2)$ convergence under assumptions of smoothness of the Euler flow.

We also obtained conditions which require only L^1 uniform integrability of tangential derivatives near the boundary, that is the requirement that for any $\epsilon > 0$ and any $L > 0$ there exists $\rho = \rho(\epsilon, L)$ such that

$$\|\partial_1 u_1^{\mathrm{NS}} \mathbf{1}_{|x_1| \le L, 0 < x_2 < \rho}\|_{L^2(0,T;L^1(\mathbb{H}))} \le \epsilon.$$

Our proofs used a boundary layer corrector differing from Kato's in its scaling properties. The boundary layer corrector u^{K} satisfies

$$\nabla \cdot u^{\mathrm{K}} = 0, \quad u_1^{\mathrm{K}}|_{\partial\mathbb{H}} = -U^{\mathrm{E}}, \quad u_2^{\mathrm{K}}|_{\partial\mathbb{H}} = 0$$

where $U^{\mathrm{E}}(x_1, t)$ is the tangential Euler velocity, assumed known and smooth. Moreover, u^{K} obeys a Prandtl $\sqrt{\nu t}$ scaling. Roughly speaking, u_1^{K} is a lift of the Euler boundary condition which obeys the heat equation $(\partial_t - \nu \partial_{x_2 x_2}) u_1^{\mathrm{K}} = 0$ to leading order in ν.

3. Nonlocal equations in bounded domains and applications

3.1. **SQG.**

The surface quasi-geostrophic equation (SQG) is a nonlocal PDE of geophysical importance. It has been extensively studied in connection with the problem of formation of singularities in incompressible fluids. The scalar inviscid equation

$$\partial_t \theta + u \cdot \nabla \theta = 0, \tag{SQG}$$

with $u = R^{\perp}\theta$ has several important analogies with the incompressible 3D Euler equations. The equation is conservative (actually Hamiltonian) and the level sets of the scalar are analogous to vortex lines in 3D incompressible Euler equations: they are carried by the flow, and their stretching is the main source of small scale formation. The problem of global regularity or finite time blow up for this equation is open. When fractional Laplacian dissipation is added to the equation, if the power of the fractional Laplacian is larger or equal to one half, then the equation has global smooth solutions. The case of power of Laplacian strictly larger than one half is a subcritical case and the global regularity of solutions follows by relatively standard methods. (Energy methods, maximum principle, interpolation). Global regularity for the critical SQG, when the power of the Laplacian equals one half, is a much more delicate matter. The nonlinearity is quadratic, and involves one derivative, and the square-root of the Laplacian gains just one derivative via elliptic estimates. Two independent proofs were obtained ten years ago, [4, 32]. Several other proofs have been found since.

In [8] we proved that the smooth solutions of critical SQG in bounded smooth domains obey a priori interior Lipschitz bounds. The critical SQG equation in bounded domains is

$$\partial_t \theta + \left(\nabla^{\perp} \Lambda_D^{-1} \theta\right) \cdot \nabla \theta + \Lambda_D \theta = 0.$$

Here Λ_D is the square-root of the Laplacian with Dirichlet boundary conditions. The proof of the interior bounds used a maximum principle, valid in bounded domains. This followed from an analogue of the Córdoba-Córdoba inequality [11]. We proved this analogue in [7] using the representation of the square root of the Dirichlet Laplacian based on the heat kernel. Our bound provided in addition a strong boundary repulsion

$$\Phi'(f)\Lambda_D f - \Lambda_D(\Phi(f)) \ge \frac{c}{d(x)} \left(f\Phi'(f) - \Phi(f) \right) \ge 0$$

where Φ is any C^2 convex function satisfying $\Phi(0) = 0$, $f \in C_0^{\infty}(\Omega)$, and $d(x)$ is the distance to the boundary. In order to bound Hölder norms we considered the evolution of finite differences $\delta_h \theta(x,t) = \theta(x+h,t) - \theta(x,t)$. In the whole space the finite differences obey evolution equations in both x and h which have a transport plus dissipation structure. In bounded domains we encountered the difficulty that the square root Laplacian is not translation invariant. The domain of definition of the Dirichlet square root Laplacian is $H_0^1(\Omega)$, and $\delta_h\theta$ might not belong to it. Thus we were lead to consider a specific family of cutoffs depending on a length scale ℓ, and first localize the equation of θ at a distance ℓ from the boundary and then perform a small (compared to ℓ) translation. Then the equation obeyed by the finite difference has the same structure as in the whole space, but for one new term, the commutator between translation and the square root of Laplacian. This is a fundamental object which we needed to bound, and it turned out that its bound is both nontrivial and unfavorable: it is the reason for the lack of uniform estimates up to the boundary. In order to obtain commutator estimates we observed an interior small time translation invariance effect in the heat kernel H for the Dirichlet Laplacian in bounded domains, which is of independent interest. The commutator estimates, used in conjunction with nonstandard bounds on the Riesz transforms and with lower bounds on the fractional Laplacian are used to prove interior C^α bounds for small α. Passing to interior Lipschitz bounds is done using a different nonlinear lower bound for the fractional Laplacian applied to derivatives of C^α functions.

3.2. **Electroconvection.** Electroconvection is the flow of fluids and particles driven by electrical forces. There are several studies of electroconvection in the physical literature, pertaining to different types of occurrences of the phenomenon. The interaction of electromagnetic fields with condensed matter is a vast and important subject, with applications ranging from solar magnetohydrodynamics to microfluidics.

We investigated a particular system, in which a charge distribution interacts with a fluid in a geometrically constrained situation. The fluid is confined to a very thin region, and a voltage difference is maintained by electrodes situated at the boundaries of the region. Physical experiments [14] and numerical studies [46] consider the flow of an annular suspended film. Despite the non-Newtonian nature of the constituent, the simplest model describes the fluid by Navier-Stokes equations confined to a fixed two dimensional region (an annulus in the cited studies). The Navier-Stokes equations are driven by body forces due to the electrical charge density and the potential. The charge density is transported by the electric potential and by the flow. The electric potential is determined by three dimensional equations, but the fluid is confined to an approximately two dimensional space. This dimensional contrast leads naturally to nonlocal equations and is one of the most natural occurrences of the Dirichlet-to-Neumann operator in fluid mechanics. In simplified situations, the system for the charge density q coupled to the fluid (u, p) is

$$\begin{cases} \partial_t q + u\cdot\nabla q + \Lambda_D q = 0 \\ \partial_t u + u\cdot\nabla u - \Delta u + \nabla p = -q\nabla\Lambda_D^{-1} q - q\nabla\Phi, \qquad \text{in } \Omega \\ \nabla\cdot u = 0, \end{cases}$$

with homogeneous Dirichlet boundary conditions for u on $\partial\Omega$. The electric potential Φ is generated by voltage applied at the boundaries of the fluid domain.

In [5] we proved that if $u_0 \in H_0^1(\Omega)\cap H^2(\Omega)\cap\mathbb{P}(L^2(\Omega))$ and $q_0 \in H_0^1(\Omega)\cap H^2(\Omega)$, then solutions to the above system exist for all time, are smooth, and uniquely determined by initial data. We proved global regularity using a two-tier approximation procedure. The fundamental property obeyed by the equation for q, a maximum principle, is essential for the global bounds. This lead us to consider a system which couples an ODE, a Galerkin approximation for u, to a PDE, the transport equation for an approximate q. Establishing existence and uniform regularity for this good approximation required an additional approximation, which however did not have a maximum principle.

Department of Mathematics, Princeton University, Washington Road, Princeton, NJ 08544
Email address: ignatova@math.princeton.edu
URL: web.math.princeton.edu/∼ignatova

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